Exploring Space Through MATH
Applications in Algebra 1

EDUCATOR EDITION

Ascent: 50 Seconds to MECO

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, Evaluate) will be used to accomplish the following objectives.

Students will
- determine independent and dependent variables;
- identify domain and range;
- interpret graphs and tables;
- identify functions; and
- use tables to determine the function rule.

Prerequisites

Students should have prior knowledge of the characteristics of functions and be able to identify functions from tables and graphs.

Background

This problem is part of a series that applies algebraic principles in NASA's human spaceflight.

The Space Shuttle Mission Control Center (MCC) and the International Space Station (ISS) Control Center use some of the most sophisticated technology and communication equipment in the world. Teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle and the ISS. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support each mission and crew during normal operations and any unexpected events.

The space shuttle is a spacecraft designed to take people and cargo into Earth's orbit. The purpose of these space shuttle missions may include such things as satellite installations and repairs, experiments, transportation of large cargo, and drop off and pick up of astronauts on the ISS. In order to perform any of its missions, the space shuttle must first lift off from Kennedy Space Center and make it into orbit. (Figure 1)

There are three main components of the space shuttle that enable it to launch into orbit. The main component is the Orbiter. It not only serves as the crew's home in space and is equipped to dock with the ISS, but it also contains maneuvering engines for finalizing the orbital trajectory. The External Tank (ET), the space shuttle's largest component, supplies the

Key Concept
Domain and range, independent and dependent variables, characteristics of a function

Problem Duration
100-110 minutes

Technology
Graphing calculator, computer with projector, video of space shuttle launch, Google Earth application (optional)

Materials
- Student Edition
- Space Shuttle Launch Video (link provided)
- Google Earth Tour SpaceShuttleAscentTour.kmz

Degree of Difficulty
Moderate

Skills
Interpret graphs and tables, identify functions and their characteristics, recognize patterns

NCTM Standards
- Number and Operations
- Algebra
- Problem Solving
- Communication
- Connections
- Representation

propellant to the Space Shuttle Main Engines (SSMEs). The third component is the Solid Rocket Boosters (SRBs). These are attached to the sides of the ET and provide the main thrust at launch. (Figure 2)

Figure 1: Space Shuttle Discovery at Liftoff

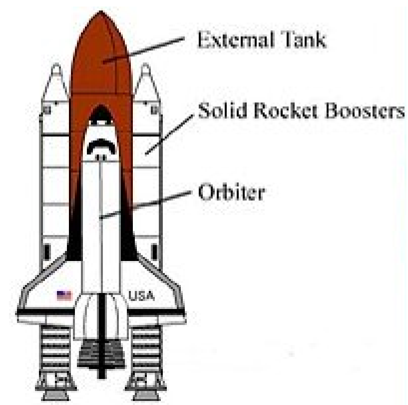


Figure 2: Main Components of the Space Shuttle

The ascent is a very dangerous and important phase of any space shuttle mission. This phase begins at liftoff and ends as the space shuttle reaches orbit. The main propulsion system (which consists of the ET and SSME) together with the SRBs supply the force needed to accelerate to the velocity of approximately 7.85 km/s that is required to attain orbit. This system is so powerful that within 1 minute the space shuttle breaks the sound barrier. As the velocity continues to increase, the stress on the space shuttle and the crew increases. Because of the immense power of its engines, the acceleration of the space shuttle must be kept below 3 *g*, or 0.03 km/s^2, to ensure the safety of the vehicle and the crew.

The space shuttle continues on its path (trajectory) until Main Engine Cut Off (MECO), soon after which it reaches orbit. It takes approximately 8 minutes and 30 seconds for the space shuttle to reach MECO, which occurs around 104 km (56 miles) above the surface of the Earth. The ET is dropped away to break up and land in the ocean, and the orbiter performs a final burn approximately 30 minutes later to reach orbital altitude of around 320 km (200 miles).

The flight controllers who have the most responsibility during the space shuttle ascent are Booster, the Propulsion Officer (PROP), and the Flight Dynamics Officer (FDO). Booster is responsible for the safe operation of the ascent propulsion systems. Once the 8.5 minutes of ascent are complete, Booster's responsibility ends. PROP is responsible for monitoring the Orbital Maneuvering Systems (OMS) and the Reactions Control Systems (RCS). These systems supplement the SSMEs and SRBs during ascent and perform final orbit insertion burns after MECO. FDO is responsible for the planning and execution of the space shuttle's trajectory during ascent, orbit, rendezvous, and re-entry.

This problem highlights the analysis of the graphs of acceleration and velocity during the 8.5 minute ascent and focuses on the 50 seconds before MECO.

NCTM Principles and Standards

Number and Operations

- Develop a deeper understanding of very large and very small numbers and of various representations of them.

Algebra

- Generalize patterns using explicitly defined and recursively defined functions.
- Understand relations and functions and select, convert flexibly among, and use various representations of them.
- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil simple cases and using technology in all cases.
- Use symbolic algebra to represent and explain mathematical relationships.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.

Problem Solving

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Communication

- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and apply mathematics in contexts outside of mathematics.
- Recognize and use connections among mathematical ideas.

Representation

- Select, apply, and translate among mathematical representations to solve problems.

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (20-30 minutes)

- Play video of the launch of STS-121. Follow the link below and choose the video entitled "The Rockets' Red Glare".
  http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/sts121/launch/sts121-allvideos.html
- Have students read the Background section together in their groups and come up with 1-2 brief summary statements from their reading.
- Open the Google Earth Space Shuttle Ascent Tour and in the left frame click on "Play me" to show students the actual trajectory of the space shuttle ascent of STS-119. See Instructions for Google Earth Space Shuttle Ascent Tour (Appendix).
- After viewing the Google Earth Tour click on "Help and Vocabulary" in the left frame to discuss NASA terms and acronyms.

2 – Explore (20 minutes)
- Distribute the worksheet, Exploring Graphs.
- Distribute the graph sheet, Ascent: 50 Seconds to MECO – Acceleration and Velocity Graphs to students. (The graph sheet can be found at the end of the lesson.)
- Arrange students in groups of 3-4 and ask them to work through questions 1-13 as a team.
- Call on students to give their answers and discuss. If available, students can use presentation technology to refer to Graph 1: Acceleration and Graph 2: Velocity.

3 – Explain (30 minutes)
- Distribute the worksheet, Interpreting Graphs & Tables.
- Ask students to work through questions 14-25 as a team.
- Facilitate student discussion and answer questions.
- Call on students to give their answers and discuss.

4 – Extend (5 minutes)
- On the worksheet, Interpreting Graphs & Tables, have students complete question 26-29 as a team.
- Call on students to give their answers and discuss.

5 – Evaluate (25 minutes)
- Take students to a computer lab, and ask them to pair up at a computer.
- Ask them to click on “Play me” in the Google Earth Tour.
- Have students work with a partner to record the required time and velocity data for Table 2 on the Interpreting Graphs & Tables worksheet. (If you only have one computer, let them record it together as a class. Call on a different student to go to the display and point out the values to record for time and velocity for each placemark.)
- On the worksheet, Interpreting Graphs & Tables, have students complete questions 30-34 with a partner.
- Call on students to give their answers and discuss.

ENGAGE
Video: The Rockets’ Red Glare

- After viewing the space shuttle ascent video, discuss with the students the major events that occur in the space shuttle ascent, (liftoff, SRB separation, MECO, ET separation, and orbit). Encourage students to share their impressions of the ascent.
- After students read the Background section and write 1-2 brief summary statements, have each group share their summaries with the class.
- Following the demonstration of the Google Earth Space Shuttle Ascent Tour, encourage student discussion using the following questions:
  - What are the major events that occur during the space shuttle ascent? (liftoff, SRB separation, MECO, ET separation, and orbit). Click on Ascent Events to demonstrate this feature.
  - What is the Mission Elapsed Time (MET) when the space shuttle reaches 100,000 ft? (1:39 min). Click on the Altitude Placemarks.
  - What is the approximate altitude of the space shuttle at MET 2:03? (150,000 ft).
  - Encourage students to share their impressions of the ascent.

EXPLORE

Exploring Graphs

Solution Key

Directions: Answer questions 1 – 13 in your group. Include units. Discuss answers to be sure everyone understands and agrees on the solutions.

Please answer the following questions about the *Graph 1: Acceleration*.

1. What variable is shown on the horizontal axis and what are the units?
   The time in seconds after liftoff.

2. Use an inequality to express the domain of the graph.
   $0 \le t \le 550$ s

3. What variable is shown on the vertical axis and what are the units?
   Acceleration in km/s^2.

4. Use an inequality to express the approximate range of the graph.
   $0 \le a \le 0.0295$ km/s^2

5. What event occurs approximately 2 minutes into the flight of the space shuttle?
   Solid Rocket Booster (SRB) Separation

6. Describe what is happening to the acceleration of the space shuttle after SRB Separation?
   It is steadily increasing.

7. At approximately 7.5 minutes into the flight what happens to the acceleration? Why?
   The acceleration is no longer increasing. The dynamic system is controlled to keep it below 3 *g*.

8. What happens to the acceleration at Main Engine Cut Off (MECO)? Explain why.
   It drops drastically to zero because the guidance computer has cut off the main engines.

Please answer the following questions about the *Graph 2: Velocity.*

9. What variable is shown on the horizontal axis and what are the units?
   The time in seconds after liftoff.

10. Use an inequality to express the domain of the graph.
    $0 \le t \le 550$ s

11. What variable is shown on the vertical axis and what are the units?
    Velocity in km/s.

12. Use an inequality to express the range of the graph.
    $0 \le v \le 7.6$ km/s

13. In the interval $0 \le t \le 510$ what appears to be happening to the velocity of the space shuttle?
    It is increasing.

EXPLAIN

Interpreting Graphs & Tables

Solution Key

Problem

The space shuttle has completed Mission STS-121. The flight controllers, FDO, Booster, and PROP, are now reviewing the space shuttle's ascent data collected by onboard computers and radars during launch. They review this data to ensure that the space shuttle performed as expected.

Directions: Answer questions 14 – 25 in your group. Include units. Discuss answers to be sure everyone understands and agrees on the solutions.

Interpreting Graphs

Refer to *Graph 1: Acceleration*.

14. Identify the independent and dependent variables with their units. Explain your choices.
    The independent variable is mission time in seconds. The dependent variable is the space shuttle's acceleration in km/s^2. Acceleration depends on time. These can also be identified by the axis on which they are graphed. The independent variable is always on the *x*-axis and the dependent variable is on the *y*-axis.

15. *Graph 1: Acceleration* clearly defines different stages of the space shuttle launch. Using *t*, write an inequality to represent the domain for the Liftoff stage through Throttle Down stage?
    $0 \le t \le 21\,\text{s}$

16. What are the coordinates of the point at which the SRBs separate? Explain specifically what the coordinates represent?
    (120, 0.008) The SRBs separate from the space shuttle at 120 seconds after liftoff and at an acceleration of 0.008 km/s^2.

17. Consider the interval from the Liftoff stage through Main Engine Cut Off (MECO).
    a. Using *t* for time in seconds after liftoff, write an inequality to represent the domain for this interval.
       $0 \le t \le 510\,\text{s}$
    b. Determine how much time elapses in minutes from liftoff to MECO.
       $$510\,\text{s} \cdot \frac{1\,\text{min}}{60\,\text{s}} = 8.5\,\text{min}$$

18. On *Graph 1: Acceleration* refer to the segment from SRB Separation to MECO.
    a. For the domain of time, $120 \le t \le 450$, what is the range? Describe what happens to the acceleration during this period.

$0.008 \le a \le 0.029 \text{ km/s}^2$

The acceleration increased smoothly from 0.008 km/s^2 to 0.029 km/s^2.

b. For the domain of time $460 \le t \le 510$, what appears to be the maximum acceleration? Why do you think the graph rises and falls repeatedly so that it almost appears horizontal?

The maximum acceleration is approximately 0.03 km/s^2. The acceleration is kept below 3 *g* to protect the crew and the space shuttle.

Refer to *Graph 2: Velocity* where *t* is time and *v* is velocity.

19. In order to exit Earth's atmosphere the space shuttle must break the sound barrier and travel faster than the speed of sound. The speed of sound at sea level is 0.34 km/s. Approximately when does the shuttle reach the speed of sound?

The space shuttle reaches the speed of sound approximately 45 seconds after liftoff.

20. Approximately how fast does the space shuttle travel after MECO (510 seconds)?

After MECO the space shuttle travels approximately 7.6 km/s.

Note: Students may have a better understanding of how fast the space shuttle is traveling if presented in customary units, i.e. approximately 17,000 miles per hour.

21. Is *Graph 2: Velocity* of the space shuttle ascent a function? Why or Why not?

Yes, *Graph 2: Velocity* is a function, because at any time, *t*, there is a unique velocity.

Interpreting Tables

Fifty seconds before MECO on STS-121 the space shuttle's velocity continues to increase. Table 1 shows the velocities during that 50 second interval.

Note: This 50 second time interval before MECO occurs 460 seconds after the space shuttle is launched and can be seen in the highlighted section on Graph 2: Velocity*. Thus time 0 in Table 1 actually occurs 460 seconds after launch.*

Table 1: Time vs. Velocity for STS-121

Time, *t* (s)	Velocity, *v* (km/s)
0	6.1
10	6.4
20	6.7
30	7.0
40	7.3
50	7.6

22. For each time interval in Table 1, find the change in velocity, *v*, and the change in time, *t.*

a. What do you notice about the changes in velocity and time?

There is a pattern for velocity and for time. The changes are constant.

b. What is the change in velocity for each interval? What is the change in time for each?

The change in velocity is 0.3 km/s. The change in time is 10 s.

c. Write the ratio of the change in *v* to the change in *t*.

$$\frac{\text{the change in } v}{\text{the change in } t} = \frac{0.3 \text{ km/s}}{10 \text{ s}}$$

d. What is another name for the ratio in part c?
Slope.

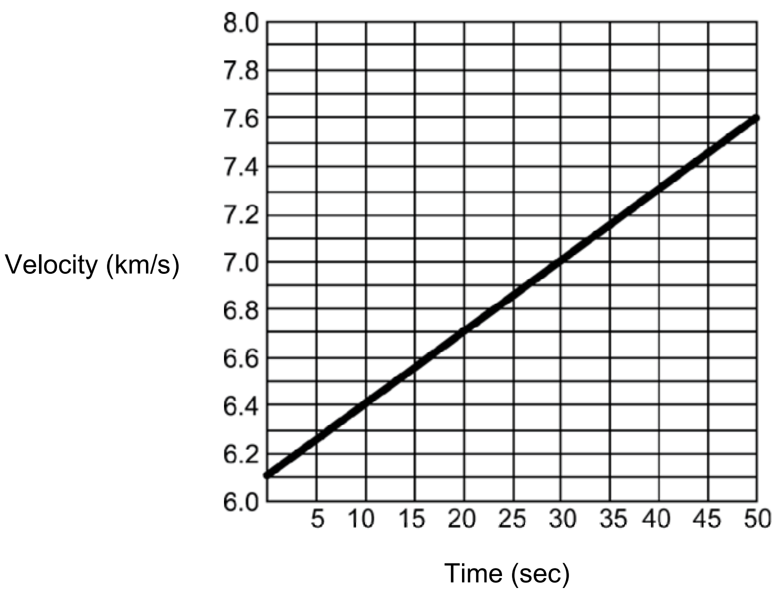
e. What is the rate of change in velocity per second? Explain your answer.

$$\frac{0.3 \text{ km/s}}{10 \text{ s}} = 0.03 \text{ km/s}^2$$

Every second the velocity increases 0.03 km/s.

23. Is the graph of the last 50 seconds before MECO a function? Why or Why not?
Yes, it is a function, because for each input value there is only one output value.

24. Using Table 1 as a reference, sketch the graph of Time vs. Velocity.



25. Use inequalities to represent the domain and range of the function in Table 1.
The domain is $0 \le t \le 50$ s
The range is $6.1 \le v \le 7.6$ km/s

EXTEND

Solution Key

Directions: Answer question 26-29 in your group. Include units. Discuss answers to be sure everyone understands and agrees on the solutions.

26. Refer to *Graph 1: Acceleration*. Use an inequality to represent the range of the graph that corresponds to the domain between “Throttle Down” and “SRB Separation”.

$0.008 < a < 0.024$ km/s^2

27. Use the data from Table 1 or the graph you sketched in question 24 to determine an equation or function rule that models the table and graph of Time vs. Velocity. Discuss your reasoning with your group.

    The equation is $v = 0.03t + 6.1$.

28. Using the calculator and the equation that you generated in question 27, determine the velocity of the space shuttle at 60 seconds and compare the value to the *Graph 2: Velocity*. Does the equation give a value that matches the graphed value? Can the equation be used after MECO. Explain why or why not.

    $v = 0.03t + 6.1$

    $v = 0.03(60) + 6.1$

    $v = 7.9$ km/s

    At 60 seconds the equation gives a velocity of 7.9 km/s which would produce the ordered pair, (520, 7.9) on *Graph 2: Velocity*. This point does not lie on the graph, but above it. The graph shows an actual velocity of only 7.6 km/s. The equation cannot be used after MECO. During the 50 seconds before MECO, the space shuttle is accelerating, but at MECO the main engines cut off, and the velocity remains constant at 7.6km/s for the remaining seconds depicted on the graph (to 550).

29. Using a graphing calculator, enter the equation that you generated in question 27, adjust the viewing window, and graph it.

    a. How does the graph compare with the sketch in question 24?

       They are the same.

    b. Access the **TABLE** screen and compare the velocity values with those in Table 1. Are the velocity values different? Explain your answer.

       They appear to be different, but the entries in the *X* column increment by only 1 second. If you scroll down to 10 seconds and compare, the velocity is 6.4 km/s, as it is in Table 1. In the **TABLE SETUP**, change the **Tbl Start** to zero and the **∆Tbl** to 10. The values match those in Table 1.

EVALUATE

Solution Key

Directions: Using the Google Earth Tour (STS-119) on a computer, work with your partner to answer questions 30 – 34. Include units.

30. Open the Google Earth Tour for STS-119 and click on “Play me”. When the tour stops, click on the red dot for “MECO” to open the dialog box. What is the Mission Elapsed Time (MET) in minutes and seconds when MECO occurs? Calculate the time at 50 seconds before MECO?

    MECO occurs at 8:24. The time at 50 seconds before MECO is 7:34.

31. Click on the box for “Velocity Placemarks” to expand the folder. Find the placemarks that fall within the domain of 50 seconds before MECO, and record the times and velocities (ft/s) in Table 2. Use the conversion factor of 1 ft = 0.0003048 km/s to convert the velocities (ft/s) in

Column 2 to velocities (km/s), and record them in Column 3. Round answers to the nearest tenth.

Note: Bold entries in a column of a table indicate the answers, and are left blank in the Student Edition.

Table 2: Google Earth Tour – Time vs. Velocity

MET (min:sec)	Velocity, *v* (ft/s)	Velocity, *v* (km/s)
7:45	**21,000**	**6.4**
7:55	**22,000**	**6.7**
8:05	**23,000**	**7.0**
8:15	**24,000**	**7.3**

32. What is the change in time for each interval? What is the change in velocity (km/s) for each interval?

    The change time is 10 seconds. The change in velocity is 0.3 km/s.

33. Find the rate of change of the velocity? Round to the nearest hundredth.

    $$\text{rate of change} = \frac{0.3 \text{ km/s}}{10 \text{ s}}$$

    $$\text{rate of change} \doteq 0.03 \text{ km/s}^2$$

34. During the 50 seconds before MECO, how does the rate of change of the velocity for STS-119 compare with the rate of change of the velocity for STS-121? Explain your answer.

    The rates of change are approximately the same, 0.03 km/s^2. This is the maximum acceleration (3 *g*) allowed in order to protect the crew and the space shuttle. During the 50 seconds before MECO, the acceleration is held just below 0.03 km/s^2.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) with the help of NASA subject matter experts and high school mathematics educators.

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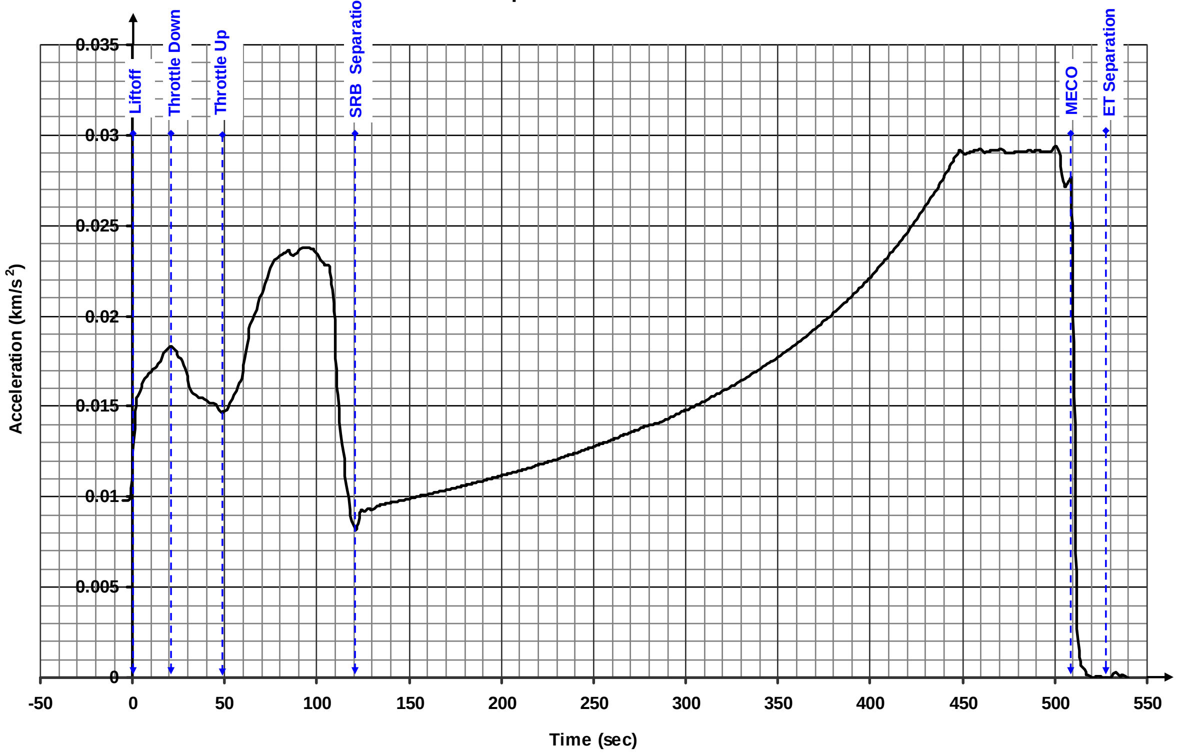
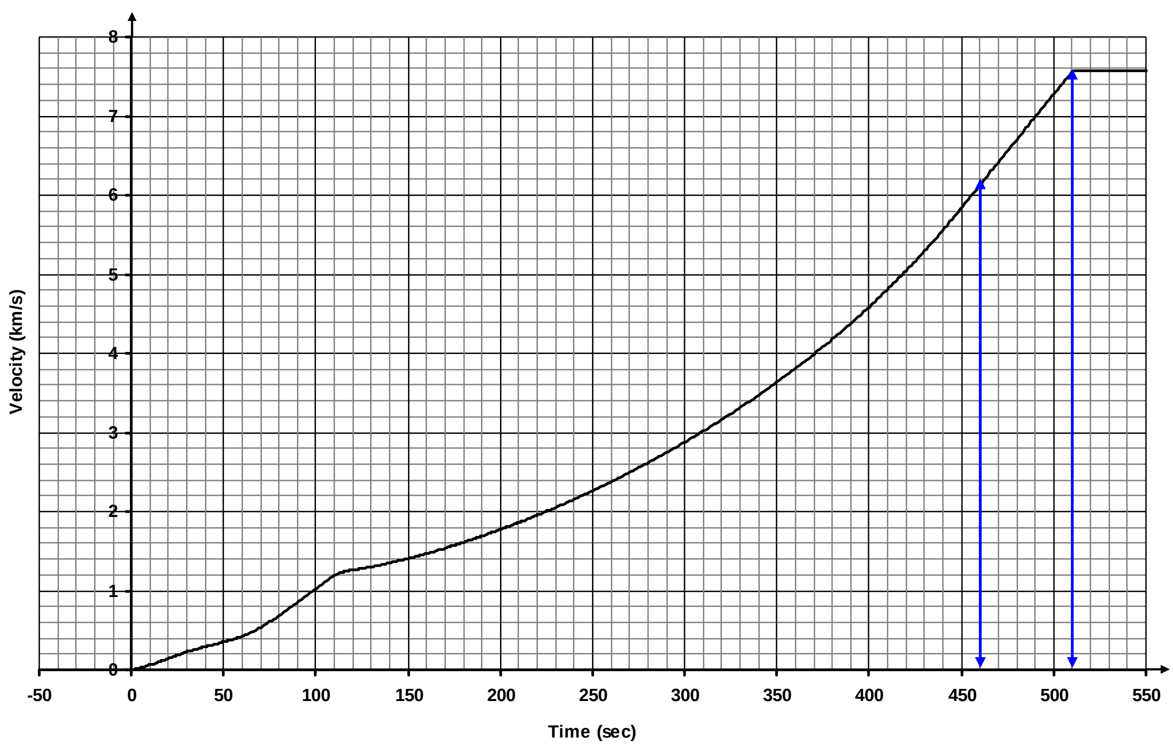
Ascent: 50 Seconds to MECO – Acceleration and Velocity Graphs

Graph 1: Acceleration



Graph 2: Velocity

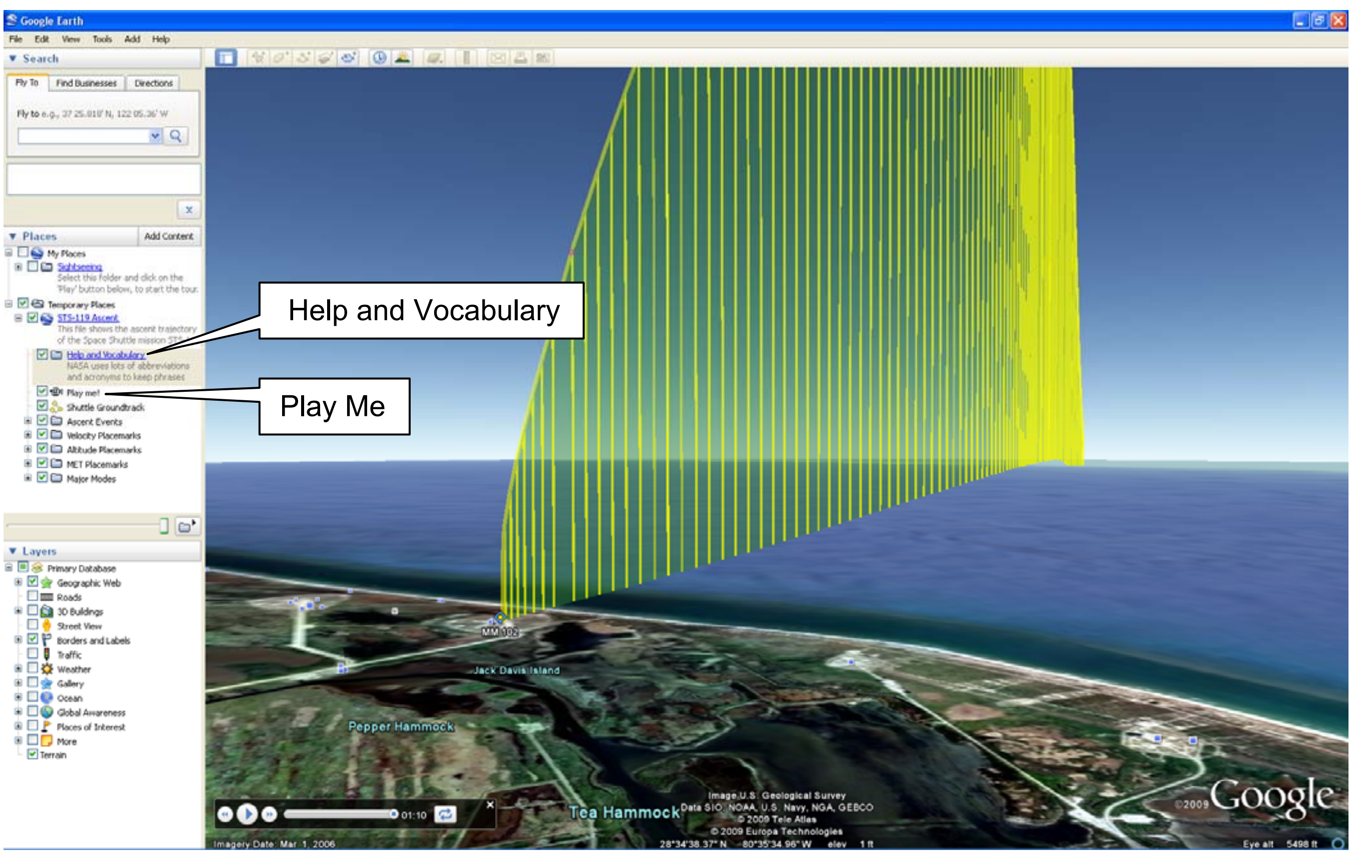


Appendix

Ascent: 50 Seconds to MECO

Instructions for Google Earth Space Shuttle Ascent Tour

1. Prior to presenting this tour, you should become familiar with the features of this file. This tour is 1:10 minutes long.
2. To access the tour, you must first have Google Earth installed on the computer. To download a free version of Google Earth, follow the link below and choose “Download Google Earth 5”.
   http://earth.google.com
3. Download the provided file SpaceShuttleAscentTour.kmz.
4. Open the file (SpaceShuttleAscentTour.kmz) on your computer by double clicking on it. Maximize the window.
5. To learn more about this file, click on **Help and Vocabulary,** located on the left frame under **Temporary Places**.
6. To start the animation, double click on “**Play Me!**”



7. The tour begins with an overview of the entire space shuttle ascent from several different angles. This overview is 20 seconds long.

8. After the overview, the sequence will begin again. There are three 5 second pauses for inserting explanations of the ascent events: SRB Separation, MECO, and ET Separation. Each event is marked with a black and red circle. At each pause, it is recommended that you click on the Pause button on the slider bar on the llower left of the screen and call on a student to describe the event and its consequences. When the student has finished, press Play to move to the next event.
9. Continue to play the animation and pause where appropriate until the tour is complete.
10. There are additional features that you may want to explore with students. You may also want to use the following subtopics to unclutter the animation, (for example, uncheck the altitude and velocity folders).

    To expand a folder in the left frame, click on the "+" next to the folder. To collapse the folder, click on the "–" next to it. For each subtopic there is a pop-up information box that can be opened by clicking on the subtopic. To close the pop-up information box, click on the "x" in the upper right corner of the box (or uncheck the small checked box to the left of the subtopic by clicking on it).

    - Ascent Events Folder (expanded)
        - Click on an event and a pop-up information box will appear on the graph.
        - Notice the bullets for the events are the same as the markers (red and black circles) that appear on the graph for these events.
    - Velocity Placemarks Folder (expanded)
        - Click on a velocity and a pop-up information box will appear on the graph.
        - Notice the bullets for the velocities are the same as the markers (green circles) that appear on the graph for these velocities.
    - Altitude Placemarks Folder (expanded)
        - Click on an altitude and a pop-up will information box will appear on the graph.
        - Notice the bullets for the altitudes are the same as the markers (pink circles) that appear on the graph for these altitudes.
    - MET (Mission Elapsed Time) Placemarks Folder (expanded)
        - Click on an MET and a pop-up information box will appear on the graph.
        - Notice the bullets for the METs are the same as the markers (orange circles) that appear on the graph for these METs.
    - Major Modes (MM) Folder (expanded)
        - Click on a major mode and a pop-up information box will appear on the graph.
        - Notice the bullets for the major modes are the same as the markers (blue diamonds) that appear on the graph for these major modes.